The base of a solid is the region in the $xy-$plane bounded by $y = f(x)$ and $y = g(x)$. SCORE: _____ / 3 PTS

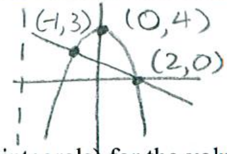
If cross sections perpendicular to the $x-$axis are equilateral triangles, the volume of the solid is 12.
Find the volume if cross sections perpendicular to the $x-$axis are semicircles.

$$\underbrace{\frac{\sqrt{3}}{4}\int_a^b s^2\,dx = 12}_{} \quad 1$$

$$\int_a^b s^2\,dx = \frac{48}{\sqrt{3}} = 16\sqrt{3}$$

$$\underbrace{\frac{\pi}{8}\int_a^b s^2\,dx}_{1} = \underbrace{2\pi\sqrt{3}}_{1}$$

Consider the region bounded by $y = 4 - x^2$ and $y = 2 - x$.



SCORE: _____ / 12 PTS

$4 - x^2 = 2 - x$

$0 = x^2 - x - 2$

$0 = (x-2)(x+1)$

$x = 2, -1$

[a] If the region is revolved around the line $x = -4$,
write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the volume of the solid

[i] using the disk or washer method **(NOTE: You do NOT need to simplify your integrand.)**

$$1\ \boxed{\int_0^3}\ \pi\ \underbrace{\left(\left(\sqrt{4-y}+4\right)^2 - (2-y+4)^2\right)}_{2} dy$$

$$1\ \boxed{+\int_3^4}\ \underbrace{\pi}_{\frac{1}{2}}\ \underbrace{\left(\left(\sqrt{4-y}+4\right)^2 - \left(-\sqrt{4-y}+4\right)^2\right)}_{2} dy$$

$y = 4 - x^2$

$x = \pm\sqrt{4-y}$

★ -2 POINTS IF YOU SWITCHED THE ANSWERS FOR [i] AND [ii]

$y = 2 - x$

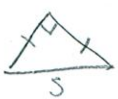
$x = 2 - y$

[ii] using the shell method **(NOTE: You do NOT need to simplify your integrand.)**

$$\frac{1}{2}\ \boxed{\int_{-1}^{2}}\ \underbrace{2\pi}_{\frac{1}{2}}\ \underbrace{(x+4)(4-x^2-(2-x))}_{2}\, dx$$

★ $-\frac{1}{2}$ POINT EACH TIME YOU FORGOT dx OR dy

[b] Suppose the region is the base of a solid.
Cross sections perpendicular to the $x-$axis are isosceles right triangles with the hypoteneuse in the base region.
Write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the volume of the solid.



$A = \frac{1}{4}s^2$

$$\frac{1}{2}\ \boxed{\int_{-1}^{2}}\ \underbrace{\frac{1}{4}}_{1}\underbrace{(4-x^2-(2-x))^2}_{1}\, dx$$

Find the area between the curves $y = 4x^2 - 7$ and $y = x^2 - 4x$ over the interval $-1 \le x \le 2$. $1\frac{1}{2}$ **SCORE: _____ / 6 PTS**

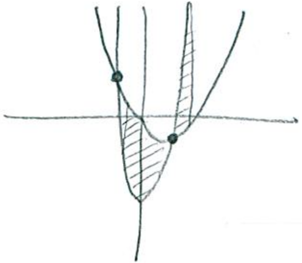
NOTE: The answer is NOT 6.

$$4x^2 - 7 = x^2 - 4x$$

$$3x^2 + 4x - 7 = 0$$

$$(x-1)(3x+7) = 0$$

$$x = 1, -\frac{7}{3}$$



$1\frac{1}{2}$ $$\int_{-1}^{1} (x^2 - 4x - (4x^2 - 7))\,dx + \int_{1}^{2} (4x^2 - 7 - (x^2 - 4x))\,dx$$

$$= \int_{-1}^{1} (-3x^2 - 4x + 7)\,dx + \int_{1}^{2} (3x^2 + 4x - 7)\,dx$$

1 1

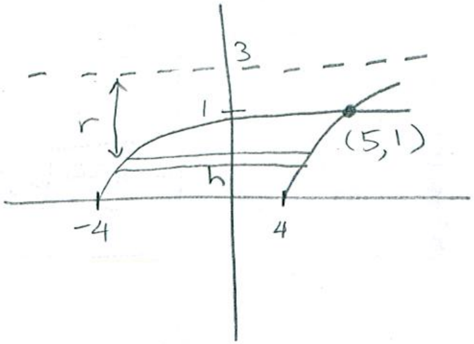
$$= (-x^3 - 2x^2 + 7x)\Big|_{-1}^{1} + (x^3 + 2x^2 - 7x)\Big|_{1}^{2}$$

$$= (4 - {}^-8) + (2 - {}^-4)$$

$$= 18$$ 1

The region bounded by $y = \frac{1}{3}\sqrt{x+4}$, $y = \sqrt{x-4}$ and $y = 0$ is revolved around the line $y = 3$. **SCORE: _____ / 12 PTS**

Find the volume of the resulting solid.



$\frac{1}{3}\sqrt{x+4} = \sqrt{x-4}$

$\frac{1}{9}(x+4) = x-4$

$x+4 = 9x-36$

$40 = 8x$

$x = 5$

$y = \frac{1}{3}\sqrt{x+4}$

$3y = \sqrt{x+4}$

$x = 9y^2-4$

$y = \sqrt{x-4}$

$x = y^2+4$

2 $\boxed{\int_0^1} \underbrace{2\pi}_{1} \underbrace{(3-y)(y^2+4-(9y^2-4))}_{4}\,dy$

$= 2\pi \int_0^1 (3-y)(8-8y^2)\,dy$

$= 2\pi \int_0^1 (24-8y-24y^2+8y^3)\,dy$

$= 2\pi \left(\underbrace{24y-4y^2-8y^3+2y^4}_{4}\right)\Big|_0^1$

$= 2\pi(14)$

$= \underline{28\pi}$ 1

DEFEND YOUR WORK
IF YOU USED DISK/WASHER
METHOD (MUCH MORE
COMPLICATED)